2.0: TRANSFORMERS

A transformer is a static device that changes ac electric power at one voltage level to ac power at another voltage level through the action of electromagnetic induction. It consist of two or more coils wrapped around a common ferromagnetic core. While these coils are (usually) not directly connected, a common magnetic flux present within the core serves as the only means of connection. They range in sizes from some of the ~~largest~~ ~~single~~ small units in electronic devices and equipments to large power transformers for converting energy of a power stations to voltage levels suitable for transmission over long distances. They may be classified according to the frequency into

(a) power frequency 50-60 c/s;

(b) Audio frequency 50 c/s – 20 kc/s;

(c) Radio frequency 20 kc/s and above

(d) pulse frequency

Alternatively, ~~they~~ the classification may be according to the purpose as

(a) Power transformer

(b) Distribution transformers

(c) Instrument transformers — (i) Voltage transformers
(ii) Current transformers.

The power transformer will be the focal point of this course. Instrument transformers will also be discussed in later sections.

2.1: THE IDEAL TRANSFORMER

An Ideal transformer is a lossless device with an Input winding and an output winding. Fig 2.1 Shows the Schematic diagram of a two winding transformer on no-load i.e. the Secondary (output) terminals ~~(out)~~ are open while the primary (input) is connected to a Source of Sinusoidal voltage of frequency f Hz.

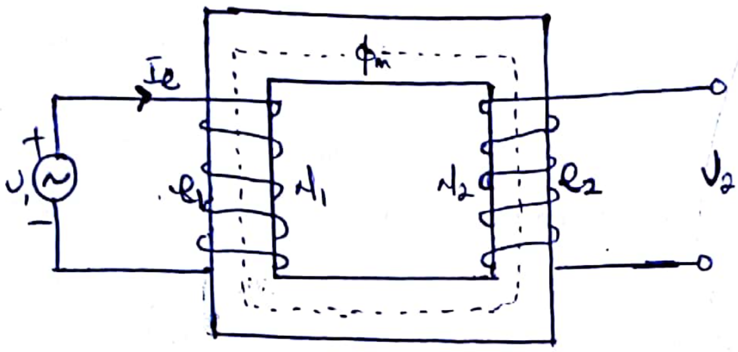

Fig 2.1: Ideal transformer with no-load

Assuming negligible winding resistance, the primary winding draws small amount of alternating current called the exciting current I_e from the voltage Source. This current establishes a flux in the core which is assumed confined to the core i.e. no leakage flux. Thus primary winding flux linkage is given by

$$\lambda_1 = N_1\phi \qquad \text{——— } 2.1$$

This induces an emf given by

$$e_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi}{dt} = V_1 \text{ (Zero resistance)} \quad \text{— } 2.2$$

therefore e_1 and ϕ must be sinusoidal of same frequency as the source V_1.

$$\phi = \phi_m \sin \omega t \qquad \text{- - - - - } 2.3$$

where ϕ_m = maximum value of core flux

$\omega = 2\pi f$ rad/s (frequency of voltage source)

eqn 2.2 can be rewritten as

$$e_1 = N_1 \frac{d\phi}{dt} = \omega N_1 \phi_{max} \cos \omega t$$

The rms value of the induced emf is

$$E_1 = \sqrt{2}\,\pi f N_1 \phi_{max} = 4.44 f N_1 \phi_{max} \quad \text{—— 2.4a}$$

Since $E_1 = V_1$ from eqn 2.2

$$\phi_{max} = \frac{E_1 (= V_1)}{4.44 f N_1} \quad \text{—— 2.4b}$$

From eqn 2.4 it ~~can be~~ is clear that the induced flux is determined by the applied voltage, frequency and the number of turns in the windings.

The same flux links ~~which~~ the primary winding with the secondary causing an induced emf of

$$e_2 = N_2 \frac{d\phi}{dt} = v_2 \quad \text{- - - - 2.5}$$

This implies that

$e_1 \propto N_1$ and $e_2 \propto N_2$

Thus, the induced emf ratio of the winding are

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \quad \text{- - - - 2.6}$$

where a is the tranformation ratio indicating the transformation action of a transformer.

The value of the excitation current i_e is such that the required mmf is produced so as to create the flux demanded by the applied voltage.

The exciting current has a magnetizing component I_m which is proportional to the sinusoidal flux and in phase with it. It also has another component I_c due to the presence of hysteresis and eddy current phenomenon which is in phase with E_1.

Thus the exciting current lags the induced emf by an angle θ_e slightly less than 90° as shown in phasor diagram of fig 2.2b. Fig 2.2a also shows the electrical circuit model of the magnetic core indicating the exciting current components.

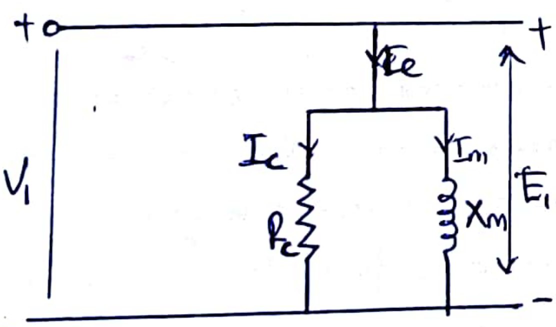

Fig 2.2a: Circuit model of Ideal Transformer on no load

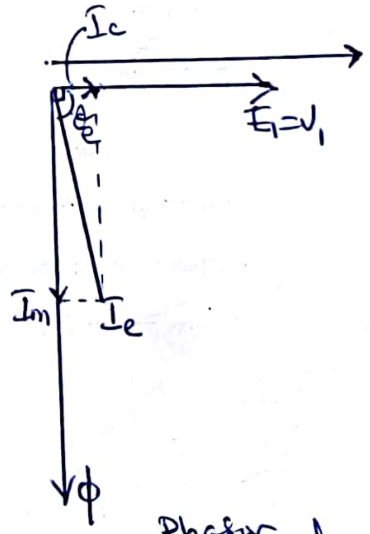

Fig 2.2b: Phasor diagram of Induced emf, flux and Exciting current.

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2.1.1: REFLECTED IMPEDANCE

Fig 2-3 shows an ideal transformer with primary winding having N_1 turns and a secondary winding with N_2 turns. In order to visualize the effect of current flowing in the secondary winding, certain assumption which are close approximation of a practical transformer are made.

~~i There is no ohmic loss and no~~

i.) The primary and secondary windings have zero resistance. It means there is no ohmic loss and no resistive votage drop in the ideal transformer.

ii) There is no leakage flux so that all the flux is confined to the core and links both winding.

iii) The core has infinite permeability so that zero magnetizing current is needed to establish the ~~required~~ required amount of flux in the core

iv) The core-loss (hysteresis and eddy-current loss) is considered zero.

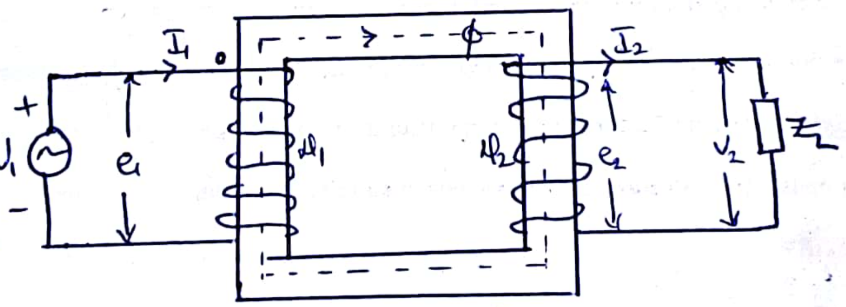

Figure 2-3: Ideal transformer on load

With the load Z_L connected to the secondary winding such that it draws an instantaneous current i_2. This current creates an mmf i_2N_2 that tend to oppose the flux ϕ. However, due to energy balance that must be maitained between the input and the output, the result is that the primary draws current

i_1 from the source to create an mmf i_1N_1 which at all times cancels out load caused by mmf i_2N_2. Thus the flux ϕ is maintained constant independent of load current.

$$i_1N_1 = i_2N_2 \qquad ---2.7$$

or

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a} \qquad ----2.8$$

From eqn 2.6 and 2.8, it follows that

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{V_2}{V_1} \qquad ---2.9$$

$$---2.10$$

or $$V_1i_1 = V_2i_2$$

This means that the instantaneous power into the primary equal instantaneous power out of the secondary (ie assumption i)

In terms of rms value

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \qquad ----2.11$$

thus

$$V_1I_1 = V_2I_2 \qquad ----2.12$$

From eqn 2.9

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad ---2.13$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad ....2.14$$

Dividing eqn 2.13 by eqn 2.14

$$\frac{V_1}{I_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_L \qquad ---2.15$$

or $$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_L = a^2 Z_L = Z_L' \qquad ---2.16$$

It can be concluded from eqn 2.16 that the impedance on the secondary side when seen (~~reffre~~ referred) on the primary side is transformed in the direct ratio of square of turns.

The equivalent circuit illustrating ~~this eqn~~ 2.16 is shown in fig 2.4a and fig 2.4b

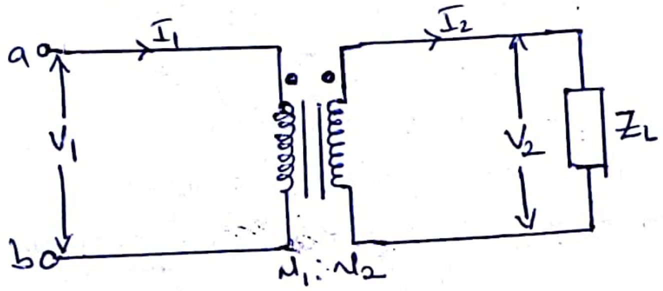

Figure 2.4a: Ideal transformer schematic

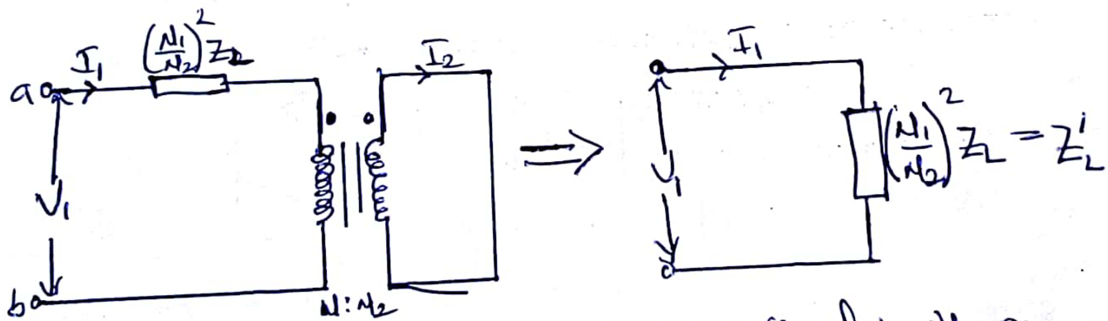

Figure 2.4b: Equivalent circuit with Z_L reffered to the primary

Similarly, impedance Z_1 referred to the secondary side is

$$Z_1' = \left(\frac{N_2}{N_1}\right)^2 Z_1 = \frac{1}{a^2} Z_1 \qquad ----2.17$$

NOTE: The process of transferring an impedance from one side of a transformer to the other is known as reffering the impedance to the other side. Also the dots marked at one end of each winding indicate the winding ends which simultaneously have the same polarity due to emfs induced.

2.12: POWER IN AN IDEAL TRANSFORMER

The power supplied to the transformer by the primary is given by the equation

$$P_{in} = V_1 I_1 \cos\theta_p \qquad \text{--- 2.18}$$

where θ_p is the angle between the primary voltage and primary current.

The power demand by the transformer secondary circuit load is given by the equation

$$P_{out} = V_2 I_2 \cos\theta_s \qquad \text{---- 2.19}$$

where θ_s is the angle between the secondary voltage and current.

Assuming $\theta_p = \theta_s = \theta$, since voltage and current angles are unaffected by an ideal transformer.

Therefore $P_{out} = V_2 I_2 \cos\theta$

Applying transformation ratio

$$V_2 = \frac{V_1}{a} \; ; \; I_2 = a I_1$$

so $P_{out} = \frac{V_1}{a} . a I_1 \cos\theta$

$$\boxed{P_{out} = V_1 I_1 \cos\theta = P_{in}} \qquad \text{---- 2.20}$$

Thus for an ideal transformer output power is equal to the input power.

Similarly for reactive pow Q and apparent power S, we have

$$\boxed{Q_{in} = V_1 I_1 \sin\theta = V_2 I_2 \sin\theta = Q_{out}.} \qquad \text{--- 2.21}$$

and

$$\boxed{S_{in} = V_1 I_1 = V_2 I_2 = S_{out}} \qquad \text{--- 2.22}$$

2.2: ~~BA~~ PRACTICAL (REAL) TRANSFORMER

An ideal transformer can of course never ~~be actually~~ actually be made. What can be produced are real transformers which contain two or more coils physically wrapped round a ferromagnetic core. To understand the operation of a real transformer, consider fig 2.5.

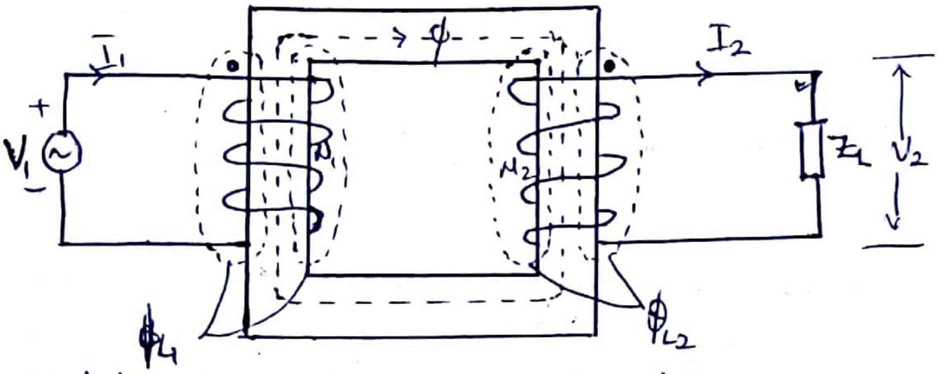

Fig 2.5: Practical transformer showing mutual and leakage fluxes of the core

For a real transformer, ~~Bo~~ both the primary and secondary windings have finite resistances r_1 and r_2 which are uniformly distributed withing the winding; giving rise to associated copper I^2R losses. While ~~the~~ the bulk of the total flux is confined to the core as mutual flux ϕ linking both the primary and secondary, a small amount of flux does leak ~~trough~~ air and link seperately the individual windings.

Transforming fig 2.5 into a semi-ideal transformer having lumped resistances r_1 and r_2 and leakage reactances given by X_{L1} and X_{L2} in series with the corresponding windings as shown in fig: 2.6

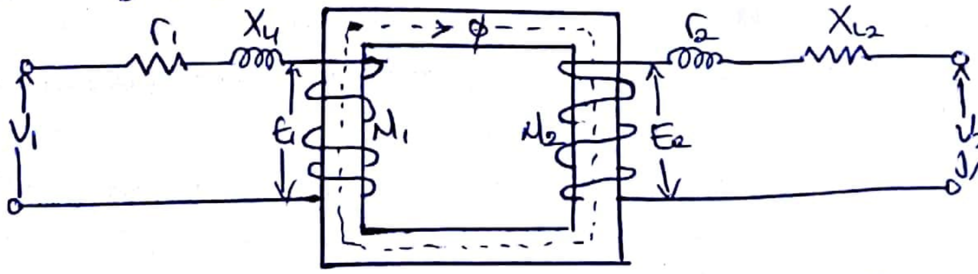

Figure 2.6: Circuit model of transformer employing semi-ideal transformer

The ~~Semi~~ induced emf of the semi-ideal transformer are E_1 and E_2 which differ respectively from the primary and secondary terminal voltage V_1 and V_2 by small voltage drops involving resistance and leakage reactances (r_1, X_{l1} and r_2, X_{l2} for primary and secondary respectively). The transformation ratio is given by

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2} \quad --- 2.23$$

But $E_1 \simeq V_1$ and $E_2 \simeq V_2$ because ~~of~~ the resistances and leakage reactances of the windings are so small.

2.2.1: REFLECTED EQUIVALENT CIRCUIT FOR A PRACTICAL TRANSFORMER

From fig 2.6 the current I_1 flowing in the primary of the semi-ideal transformer can be ~~is cont~~ visualized to comprise of two components as:

i) Exciting current I_e whose magnetizing component I_m creates the mutual flux ϕ and whose core-loss component I_c provides the loss associated with alternation of flux.

ii) A load component I_1' which counterbalances the secondary mmf I_2N_2 so that the mutual flux remains constant independent of load, determined only by E_1. Thus

$$I_1 = I_1' + I_e \quad --- 2.24$$

where

$$\frac{I_1'}{I_2} = \frac{N_2}{N_1} \quad --- 2.25$$

~~As discussed earlier,~~

~~Reffering the components of the secondary windings~~ ~~state of the~~

The corresponding equivalent circuit modelling the behavior of a real transformer is shown in fig 2.7

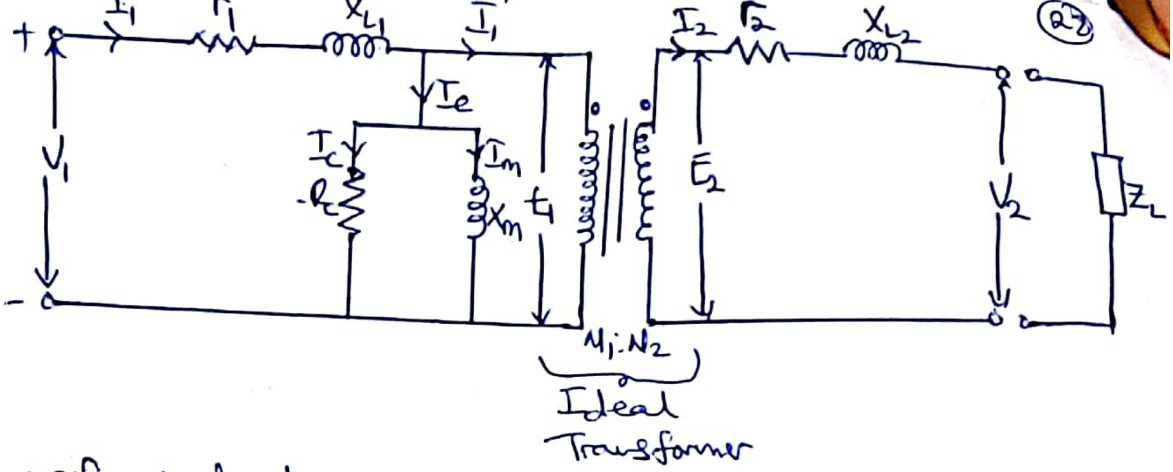

Figure 2.7: Equivalent circuit of a practical transformer

Referring the impedance of the secondary side of the ~~Ideal~~ equivalent circuit to the primary side results in figure 2.8 wherein

$$X_{L2}' = \left(\frac{N_1}{N_2}\right)^2 X_{L2} \qquad \text{---- } 2.26a$$

$$r_2' = \left(\frac{N_1}{N_2}\right)^2 r_2 \qquad \text{---- } 2.26b$$

If a load Z_L is connected, the referred load impedance is given by

$$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L \qquad \text{---- } 2.26c$$

The load voltage and current reffered to the primary side are

$$V_2' = \left(\frac{N_1}{N_2}\right) V_2 \qquad \text{----- } 2.27a$$

$$I_2' = \left(\frac{N_2}{N_1}\right) I_2 \qquad \text{----- } 2.27b$$

Therefore there is no need to show the ideal transformer reducing the transformer equivalent circuit to the T-circuit of fig 2.8 as ~~reffe~~ referred to side 1.

(29)

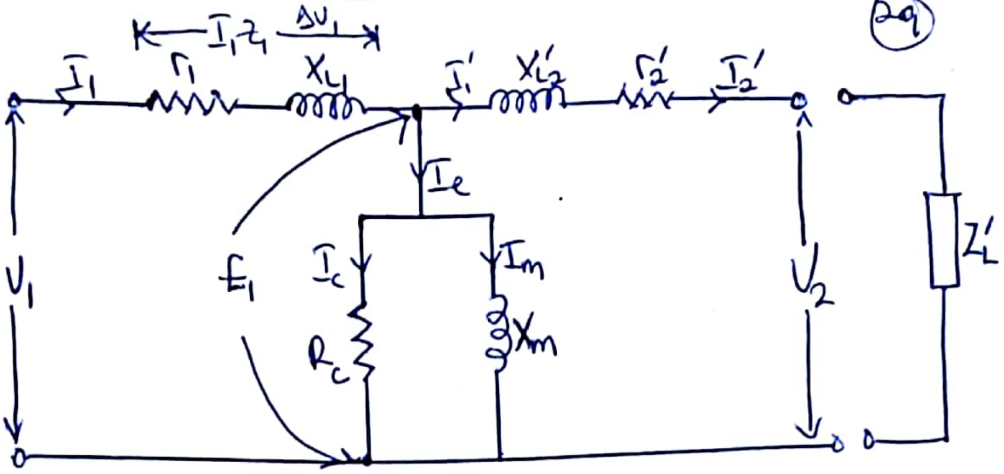

Figure 2.8: Referred equivalent circuit of practical transformer

For an unloaded (open circuit) condition, $I_1'=0$ and $I_1 = I_e$. This produces a small voltage drop ($\Delta V_1 = I_1 Z_1$) due to Z_1.

$$\Rightarrow V_1 \gg \Delta V_1$$

and $\Delta V_1 = f(I_1, Z_1)$, thus approximate circuit can be obtained by moving the r–L branch across ΔV_1. The resulting circuit becomes

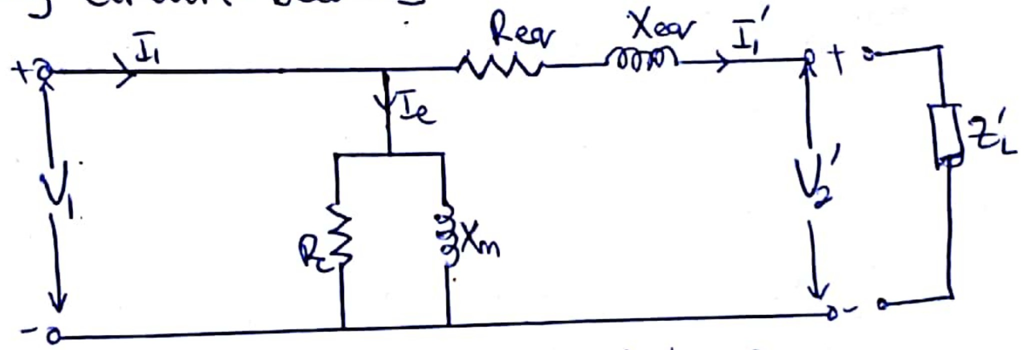

Figure 2.9: Approximate equivalent circuit of transformer

In computing voltages from the approximate equivalent circuit, the parallel magnetizing branch has no role to play and can, therefore be ignored as in fig 2.10.

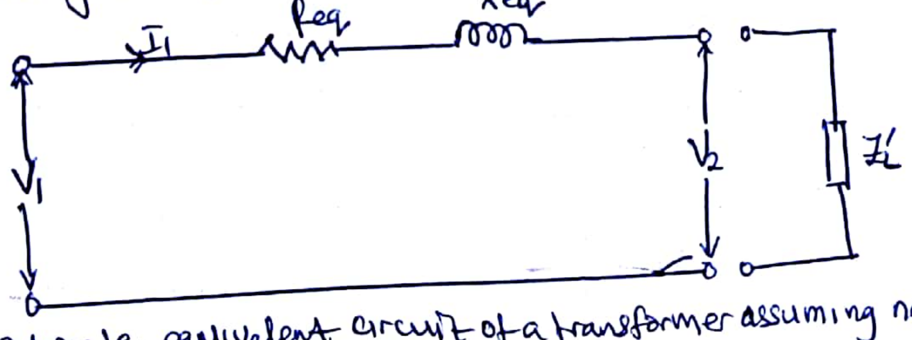

Figure 2.10: Approximate equivalent circuit of a transformer assuming negligible ~~magnetiz~~ magnetizing branch current.

From fig 2.10, the quantities of the approximate equivalent circuit are

$$R_{eq} = r_1 + a^2 r_2 \qquad --- 2.28a$$

$$X_{eq} = X_{L1} + a^2 X_{L2} \qquad --- 2.28b$$

$$Z_{eq} = R_{eq} + jX_{eq} \qquad --- 2.28c$$

Further evaluation of the primary current depending on the nature of the load.

$$\therefore\ I_1 = \frac{V_1}{Z_{eq} + Z_L'} = \frac{V_1}{(R_{eq} + jX_{eq}) + \alpha^2 (R_L + jX_L)} \qquad -- 2.29$$

Assignment

Draw the phasor diagram for the practical transformer of fig. at a leading, lagging and unity powerfactor. In each, show the relation between V_1, V_2, I_q and I_2 $I_1 Z_{eq}$, $I_1 R_{eq}$ are reflected to the primary circuit.

2.3: TRANSFORMER LOSSES

The various losses in a transformer are outlined as follows:

CORE-LOSS: ~~On~~ Hysteresis and eddy-current losses resulting from the alternating magnetic flux make up the core-loss. It may be emphasized here that the core-loss is constant for a transformer operated at constant voltage and frequency. ~~This is~~ Same can be said of all power frequency equipments.

Copper-loss (I^2R loss): This occurs in winding resistance when the transformer carries the load current; It varies as the square of the loading expressed as a ratio of the full-load.

Load (stray) loss: This is largely due to the leakage fields inducing eddy-currents in the wall of the transformer tank and conductors.

Dielectric-loss: this loss is as a result of the insulating materials, particularly the oil and solid insulations.

2.4: TRANSFORMER TESTING.

Two reasons which do not warrant the testing of large transformers by direct load test are:

i) Large amount of energy has to be wasted in such a test

ii) It would be an impossible task to arrange a load large enough for direct loading.

However, it is possible to experimentally determine the values of the reactances and resistances in the transformer model. An adequate approximation of these values can be obtained with the open-circuit test and the short-circuit test.

2.4.1: OPEN-CIRCUIT TEST

The purpose of this test is to determine the shunt branch parameters of the equivalent circuit of the transformer. One of the windings (usually the LV side) is connected to supply rated voltage while the other winding (HV side) is kept on open circuit; this is for convinience and availability of supply. Fig 2.11 illustrates this setup during the test. The metering is arranged to read as follows:

Voltmeter $= V_1$; Ammeter $= I_1$; and Wattmeter $= P_{oc}$.

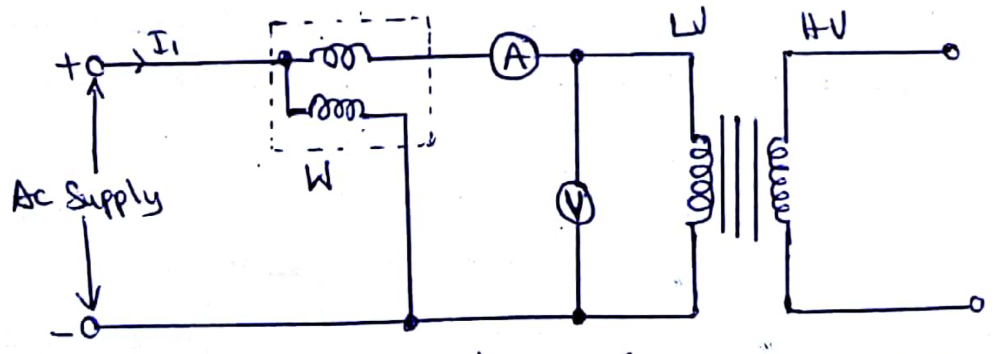

Figure 2.11: Connection digram for open circuit test

~~From fig~~

~~The series impedance of fig acts so small and can be neglected~~

By neglecting the series impedance of fig 2.11 due to its small value, V_1 can be regarded as $= E_1$. This means that for all practical purposes the power input on open circuit equals the core (iron) loss ie

$$P_{oc} = P_{i}(\text{iron-loss}) \quad \text{--- } 2.30$$

The value of the shunt parameters (R_c & X_m) is easily determined by converting them in to admitance. Thus the conductance of the core-loss resistor is given by

$$G_c = \frac{1}{R_c} \quad \text{--- } 2.31$$

and susceptance of the magnetizing ~~induc~~ recatance is

$$B_m = \frac{1}{X_m} \quad \text{--- } 2.32$$

total admitance is given by

$$Y_E = G_c \mp jB_m = \frac{1}{R_c} \mp j\frac{1}{X_m} \quad \text{--- } 2.33$$

Now

$$I_{oc} = V_{oc} Y_E = V_1 Y_E = I_1 \quad \text{--- } 2.34a$$

$$Y_E = \frac{I_1}{V_1} \quad \text{--- } 2.34b$$

But

$$P_{oc} = V_1^2 G_c \quad \text{or} \quad G_c = \frac{P_{oc}}{V_1^2} \quad \text{--- } 2.35$$

therefore

$$B_m = \sqrt{Y_E^2 - G_c^2} \quad \text{--- } 2.36$$

The power factor can also be determined from

$$P_{oc} = I_{oc}V_{oc}\cos\theta = I_1 V_1 \cos\theta \quad \text{--- } 2.37$$

$$\theta = \cos^{-1}\left(\frac{P_{oc}}{I_1 V_1}\right) \quad \text{--- } 2.38$$

... ed as below: ... pedance

These values are as referred to the side from which the test is conducted and could be referred to the other side if so desired by the inverse square of transformation ratio.

It is, therefore seen that the open-circuit test yields the values of the core-loss and shunt parameters of the equivalent circuit.

2.4.2: SHORT CIRCUIT TEST

This test is conducted to determine the values of the series parameters of a transformer. Due to the nature of voltage and current to be handled, the test is usually conducted from the HV side of the transformer while the LV side is short-circuited. Since the transformer impedance is very small, the voltage V_{sc} needed to circulate the full-load current under short circuit is as low as 5-8% of rated voltage. Thus negligible current flows through the shunt branch of the circuit. Fig 2.12 shows the circuit setup during the test. The meter readings under short circuit procedure are: Voltmeter $= V_{sc}$; Ammeter $= I_{sc}$; Wattmeter $= P_{sc}$

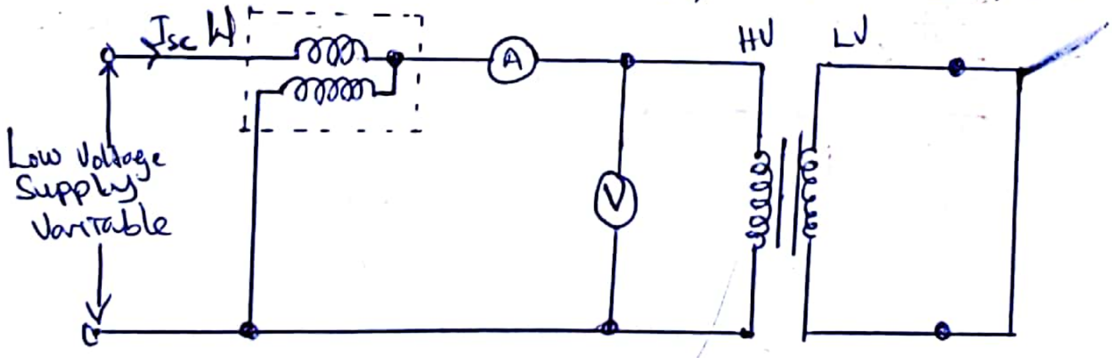

Figure 2.12: Short-circuit test setup on transformer.

With the shunt branch left out, the power input corresponds only to the copper-loss, i.e.

$$P_{sc} = P_c \text{(Copper-loss)} \quad \text{---- } 2.39$$

From the equivalent circuit of fig equivalent series impedance is computed as below:

$$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \sqrt{R_{eq}^2 + X_{eq}^2} \qquad \text{---}2.40$$

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Equivalent resistance, $R_{eq} = \frac{P_{sc}}{(I_{sc})^2}$ ---2.41

Equivalent reactance, $X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$ ---2.42

Recall that: $R_{eq} = r_1 + r_2'$

and $X_{eq} = X_{L1} + X_{L2}'$

~~Note also that these values~~
Since these values are referred to the HV side from which the test is conducted, If the values desired are for the LV side, they could be easily refered as such.

Example: The following data were obtained on a 20kVA, 50Hz, 2000/200V distribution transformer:

OPEN CIRCUIT ON LV SIDE: $V_{oc} = 200V$, $I_{oc} = 4A$, $P_{oc} = 120W$

SHORT CIRCUIT ON HV SIDE: $V_{sc} = 60V$, $I_{sc} = 10A$, $P_{sc} = 300W$

Draw the approximate equivalent circuit of the transformer referred to the HV and LV sides respectively.

Solution:

For open circuit test:

$$P.f = \cos\theta = \left(\frac{P_{oc}}{V_{oc}I_{oc}}\right)$$

$$= \cos\theta = \frac{120}{200 \times 4} = 0.150 \text{ lagging}$$

Excitation admittance is given by:
$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} p.f$$

$$= \frac{4}{200} \angle -\cos^{-1} 0.150 = 0.02 \angle -81.425$$

$$= 0.003 - j0.0198 = \frac{1}{R_c} - j\frac{1}{X_m}$$

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Therefore : $R_c = \dfrac{1}{0.003} = \cancel{7.5\,\Omega}\ \ 333\,\Omega$

$X_m = \dfrac{1}{0.0198} = \cancel{49.4\,\Omega}\ \ 50.5\,\Omega$

From the Short circuit test.

$$P.f = \cos\theta = \frac{P_{sc}}{V_{sc} I_{sc}} = \frac{300}{60 \times 10} = 0.50 \text{ lagging}$$

The Series impedance is given by

$$Z_{eq} = \frac{V_{sc}}{I_{sc}} \angle -\cos^{-1} p.f$$

$$= \frac{60}{10} \angle -60^\circ$$

$$= 6.0 \angle -60^\circ = 3.0 - j5.2\ \Omega$$

Therefore: $R_{eq} = 3.0\,\Omega$; $X_{eq} = 5.2\,\Omega$

Transformation turns ration $\dfrac{N_H}{N_L} = \dfrac{V_1}{V_2} = \dfrac{2000}{200} = 10$

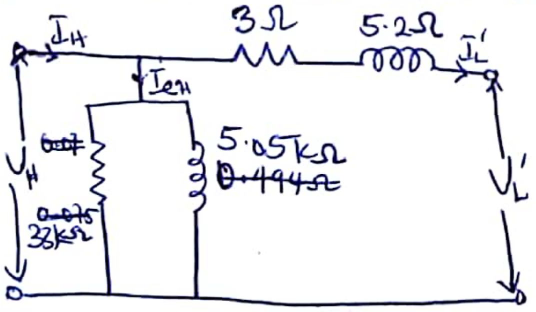
equivalent circuit parameters referred to the HV side:

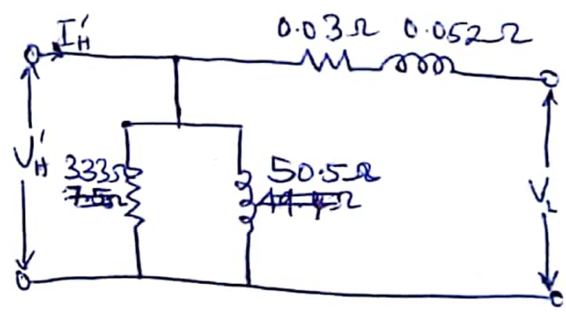
$R_c = 333 \times 10^2 = 33.3\,k\Omega$; $\cancel{\dfrac{7.5}{10^2}} = \cancel{0.075\,\Omega}$; $X_m = 50.5 \times 10^2 = 5.05\,k\Omega$; $\cancel{\dfrac{49.4}{10^2}} = \cancel{0.494\,\Omega}$

equivalent circuit parameters refered to the LV side;

$R_{eq} = \dfrac{3}{10^2} = 0.03\,\Omega$; $X_{eq} = \dfrac{5.2}{10^2} = 0.052\,\Omega$

The equivalent circuits ~~are~~ shown in





2.5: TRANSFORMER EFFICIENCY AND VOLTAGE REGULATION.

Power and distribution transformers are designed to operate under conditions of constant rms voltage and frequency and so the efficiency and voltage regulation are of prime importance.

2.5.1: EFFICIENCY:

The rated capacity of a transformer is defined as the product of the rated voltage and full load current on the output side. The efficiency η of a transformer is defined as the ratio of the useful power to the input power. Thus

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \quad \text{---} 2.43$$

The best and accurate method of determining efficiency would be to find the losses from the OC and SC tests. With this data efficiency can then be calculated as.

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% \quad \text{---} 2.44$$

Copper loss in the two windings are

$$\left.\begin{aligned} P_{cu} &= I_1^2 r_1' + I_2^2 r_2 \\ &= I_2^2 R_{eq}' \end{aligned}\right\} \text{---} 2.45$$

Where R_{eq}' is the equivalent resistance referred to the secondary side. Thus it is found that the copper losses is proportioned to the square of the load current.

Recall from eqn 2.19 that transformer output power

$$P_{out} = V_2 I_2 \cos\theta_s \quad \text{---} 2.46$$

where $\cos\theta_s$ = load pf.

from eqn 2.44

$$\eta = \frac{V_2 I_2 \cos\theta_s}{V_2 I_2 \cos\theta_s + P_i + I_2^2 R_{eq}'} \quad \text{---} 2.47$$

For maximum efficiency

$$P_i = I_2^2 R'_{eq} \quad \text{---- 2.48}$$

ie Copper loss (variable) = Core-loss (constant), thus

$$I_2^2 = \frac{P_i}{R'_{eq}} \quad \text{--- 2.49}$$

A family of efficiency curves can be obtained by varying the load current and power factor as shown in figure 2.13

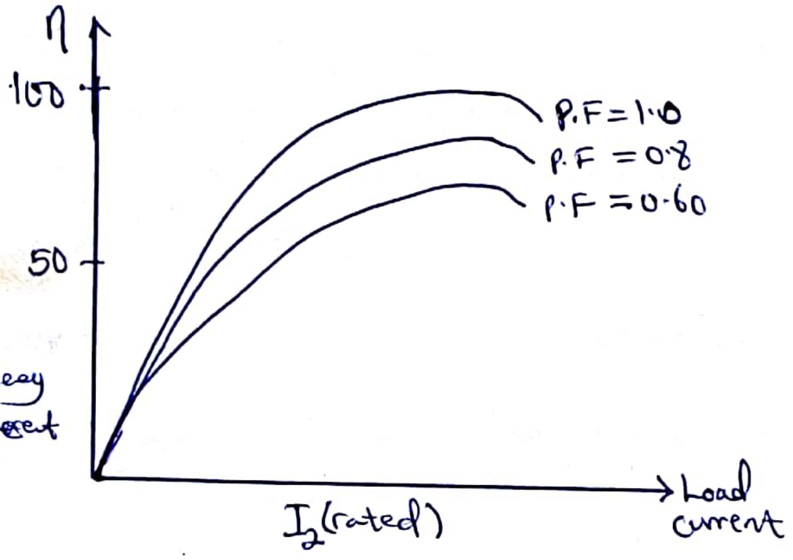


Fig 2.13: Efficiency Curves at different Power factor

2.5.2: VOLTAGE REGULATION

The figure of merit which determines the voltage drop characteristics of a transformer is the voltage regulation. It is defined as the change in magnitude of the secondary voltage, when full-load (rated load) of specified power factor supplied at rated voltage is turned off. ie reduced to no-load with primary voltage (and frequency) held constant, as percentage of the rated load terminal voltage.

$$VR\% = \frac{V_1 - V_2'}{V_{1(rated)}} \times 100\% \quad \text{or} \quad VR\% = \frac{V_1' - V_2}{V_2} \times 100\% \quad \text{--- 2.50}$$

Fig 2.14a Shows the equivalent circuit referred to the Secondary side and fig 2.14b gives its phasor diagram.

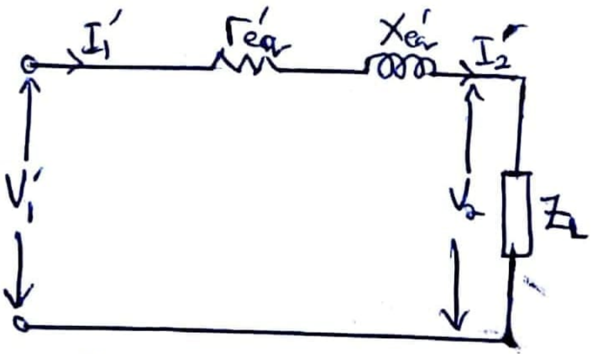


Fig 2.14a

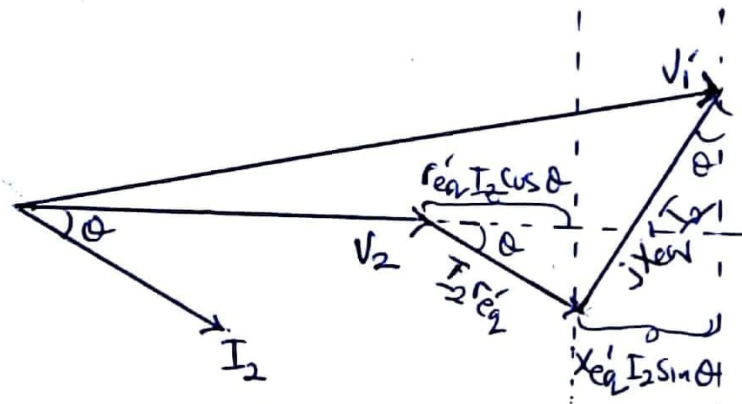


Fig 2.14b

From the phasor diagram,

$$\bar{V}_1' = \bar{V}_2 + (r_{eq}' + jX_{eq})\bar{I}_2$$

$$V_1' - V_2 \simeq (r_{eq}' + jX_{eq})I_2 = I_2 r_{eq}' \cos\theta + I_2 X_{eq}' \sin\theta$$

therefore $$VR\% = \frac{I_2 r_{eq}' \cos\theta + I_2 X_{eq}' \sin\theta}{V_2} \times 100\% \quad \text{---} 2.51$$

Example: A 15kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the primary side of the transformer:

OPEN CIRCUIT TEST: $V_{oc} = 2300V$; $I_{oc} = 0.21A$; $P_{oc} = 50W$

SHORT CIRCUIT TEST: $V_{sc} = 47V$; $I_{sc} = 6.0A$; $P_{sc} = 160W$

Find: ~~the~~

a) the equivalent circuit of this transformer reffered to the high voltage side

b) the equivalent circuit of this transformer referred to the low voltage side.

c) the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor and 0.8 leading power factor

d) the efficiency of the transformer at full load with power factor of 0.8 lagging?

Solution:

a) From the open circuit test, impedance angle is

$$\cos\theta_{oc} = \frac{P_{oc}}{I_{oc}V_{oc}} = \frac{50}{(2300)(0.21)} = 0.1035$$

$$\theta = \cos^{-1}(0.1035) = 84^\circ$$

$$Y_E = \frac{I_{oc}}{V_{oc}}\angle{-84^\circ} = \frac{0.21}{2300}\angle{-84^\circ} = 9.13\times10^{-5}\angle{-84^\circ}\ \mho$$

$$Y_E = (0.0000095 - j0.0000908)\ \mho$$

$$R_c = \frac{1}{0.0000095} = 105\,k\Omega$$

$$X_m = \frac{1}{0.0000908} = 11\,k\Omega$$

From the short circuit test data,

$$\theta_{sc} = \cos^{-1}\frac{P_{sc}}{V_{sc}I_{sc}} = \cos^{-1}\frac{160}{(47)(6)}$$

$$\theta_{sc} = 55.4^{\circ}$$

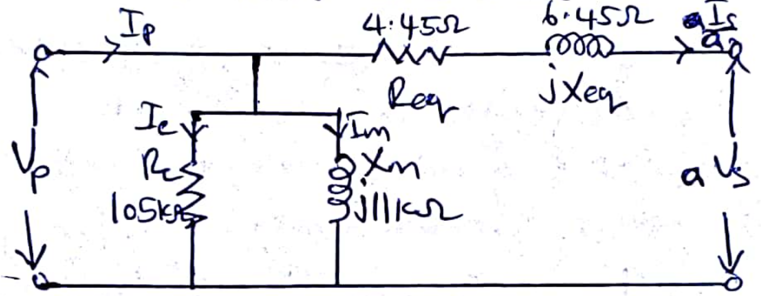
The series equivalent impedance is thus

$$Z_{SE} = \frac{V_{sc}}{I_{sc}}\angle\theta_{sc} = \frac{47}{6}\angle 55.4\,\Omega$$

$$= 7.833\angle 55.4 = 4.45 + j6.45$$

ie $R_{eq} = 4.45\,\Omega$; $X_{eq} = 6.45\,\Omega$

The equivalent circuit is shown in



b) To Find the equivalent circuit referred to the low voltage side:

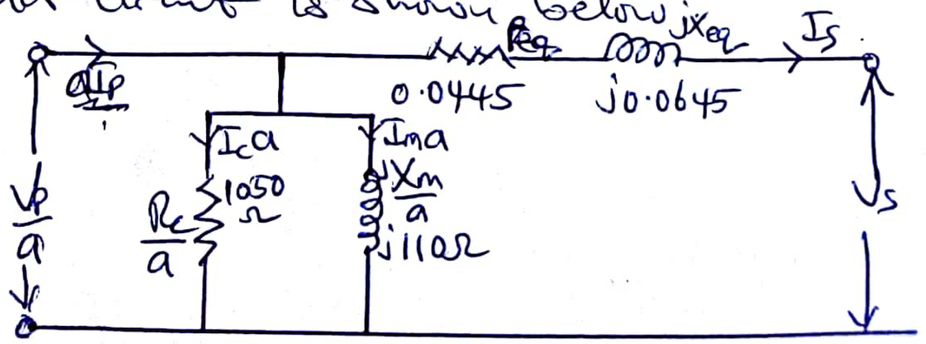
$$a = \frac{N_p}{N_s} = \frac{2300}{230} = 10$$

The parameters of the equivalent circuit calculated in a) is devided by a^2 and is given by

$$R_c = \frac{105\times10^3}{10^2} = 1050\,\Omega \;;\; X_m = \frac{11\times10^3}{10^2} = 110\,\Omega$$

$$R_{eq} = \frac{4.45}{10^2} = 0.0445\,\Omega \;;\; X_{eq} = \frac{6.45}{10^2} = 0.0645\,\Omega$$

The equivalent circuit is shown below



c) The full-load current on the secondary side of the transformer is

$$I_{s(rated)} = \frac{S_{rated}}{V_{s(rated)}} = \frac{15000}{230} = 65.2A$$

At ~~cos~~ Pf = 0.8 lagging ie $\cos\theta = 0.8$
then $\sin\theta = 0.6$

from $VR\% = \frac{I_2 (r_{eq}\cos\theta + X_{eq}\sin\theta)}{V_2} \times 100\%$

$$= \frac{65.2(0.0445 \times 0.8 + 0.0645 \times 0.6)}{230} \times 100\%$$

$$= 2.1\%$$

At Pf = 1.0, ie $\cos\theta = 1$, then $\sin\theta = 0$

$$VR\% = \frac{65.2(0.0445 \times 1)}{230} \times 100 = 1.26\%$$

At pf = 0.8 leading ie $\cos\theta = 0.8$, $\sin\theta = 0.6$

$$VR\% = \frac{65.2(0.0445 \times 0.8 - 0.0645 \times 0.6)}{230} \times 100\%$$

$$= -0.088\%$$

d) To find the efficiency of the transformer, the losses are first computed

Copper loss $P_{cu} = I_s^2 R_{eq} = (65.2)^2(0.0445) = 189W$

Core loss $P_{core} = \frac{V_p'^2}{R_c} = \frac{(234.85)^2}{1050} = 52.5W$

But $P_{out} = V_s I_s \cos\theta = (230)(65.2)\cos 36.9 = 12,000$

therefore $\eta = \frac{V_s I_s \cos\theta}{V_s I_s \cos\theta + P_{cu} + P_{core}} \times 100\%$

$$= \frac{12000}{189 + 52.5 + 12,000} \times 100\% = 98.03\%$$

2.6: CONSTRUCTIONAL FEATURES OF TRANSFORMER

A typical transformer consist of the following parts.

1) CORE: This is made up of laminated iron sheets cut to shape. The purpose of laminating the core is to reduce the iron loss due to eddy currents induced by the alternating flux.

2) WINDINGS: Conductors windings around the core. ~~transmit~~ convey electrical energy to or from a working region. The primary function of the windings is to induce emf in the transformer.

3) INSULATING MEDIA: The winding of huge transformer ~~use~~ use conductors with heavy insulation such as paper, cloth etc. In addition, the built-up transformer is usually immersed in a tank filled with ~~non~~ insulating oil called transformer oil.

Transformers ~~cooled~~ can be classified by the type of cooling method employed. Common cooling methods are:

ⓐ Oil feed self cooled

ⓑ Oil feed water cooled

ⓒ Air blast

2.6.1: Oil feed Self cooled: This is common to small and medium size distribution transformers. The oil here is a medium used for cooling (ie coolant). Heat is taken by the oil from the core and windings which is then transfered from the container to the surrounding air. The external surface of each transforme varies proportionally to the capacity of such transformer. This is to provide enough surface area for air circulation.

2.6.2 Oil feed water cooled: This is used for high voltage transmission line. The coil and core are insoted in oil while pipes are located closed to the surface. Cold water is circulated through the pipe and this is used to extract the heat from the surface of the pipe.

2.6.3 Air blast: This type is used at voltages below 25kV. ~~Attct~~ Medium of Cooling is air. This is achieved by putting the transformer in a thin box opened at both ends. Air is blown from bottom to the top by means of a fan or blower ~~Now~~

Structurally, there are two principal types of Construction used for Single-phase transformer. These are the "core" and "Shell" type.

2.6.4: CORE TYPE

In the core type, the windings are made to surround the iron in the fashion illustrated by fig 2.15. In the typical configuration, one-half the turns of low voltage coil are placed on each leg nearest the iron separated by an appropriate layer of insulation. Then following another layer of suitable insulation half the turns of high-voltage windings are placed over low-voltage coil. The process is repeated for the Second leg as well. The core type construction is favoured for high voltage transformers because the problem of insulation is easier.

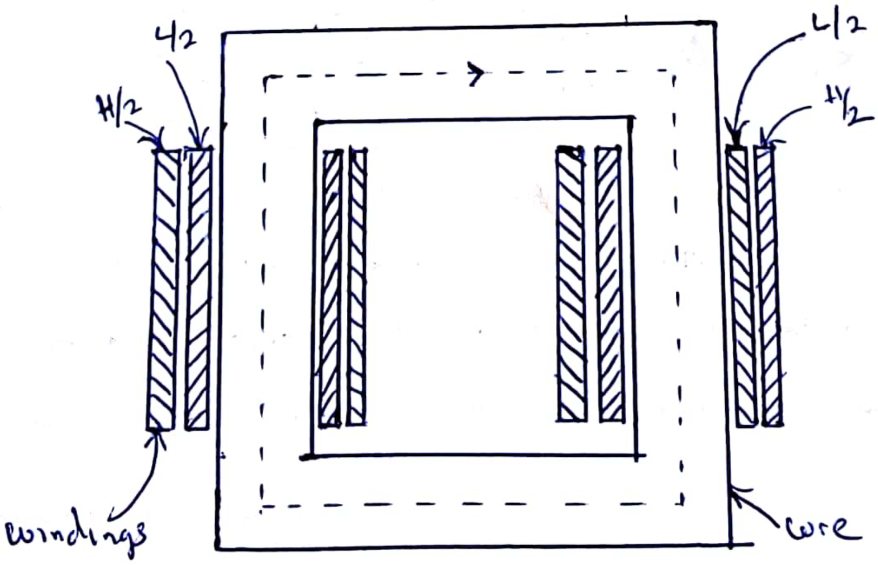


Fig 2.15: Core type transformer

H denotes the number of turns of the high voltage coil; L denotes the number of ~~winding~~ turns of the low voltage coil.

2.6.5: THE SHELL TYPE

In the Shell type transformers, it's is iron that appears to surround the coils as shown in fig 2.16. The coils in this configuration are generally of a pancake form as distinguised from the cylindrical forms used in the core-type transformers. Often the low-Voltage winding is devided into three Section with the two outer Scetions each having a quater of the turns and the remaining half is Sandwitched between the two halves of the high voltage windings as indicated in the figure.

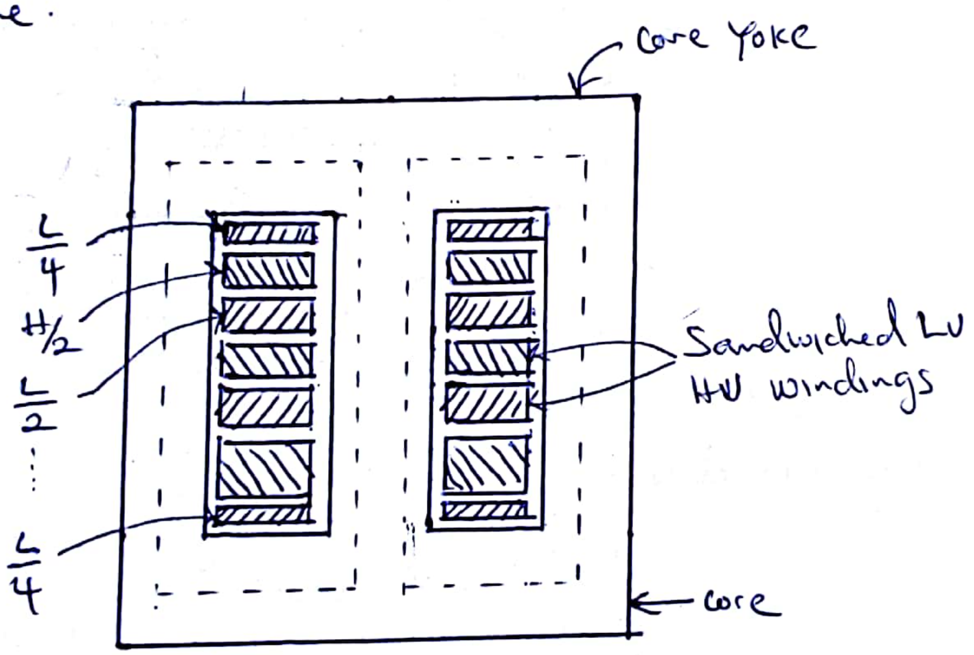


Fig 2.16: Shell type transformer

The Shell-type construction is favoured for power transformers where large currents are made to flow. The Iron shell provides better mechanical protection to the windings in such circumstances.

2.7: AUTO-TRANSFORMER

This is a simple winding transformer having a part of it's winding common to the primary and secondary circuit. Fig 2.17 shows the single-phase autotransformer having N_1 turns primary with N_2 turns tapped for a ~~circuit~~ lower voltage secondary. The winding section BC of N_2 turns is common to both primary and secondary circuits,

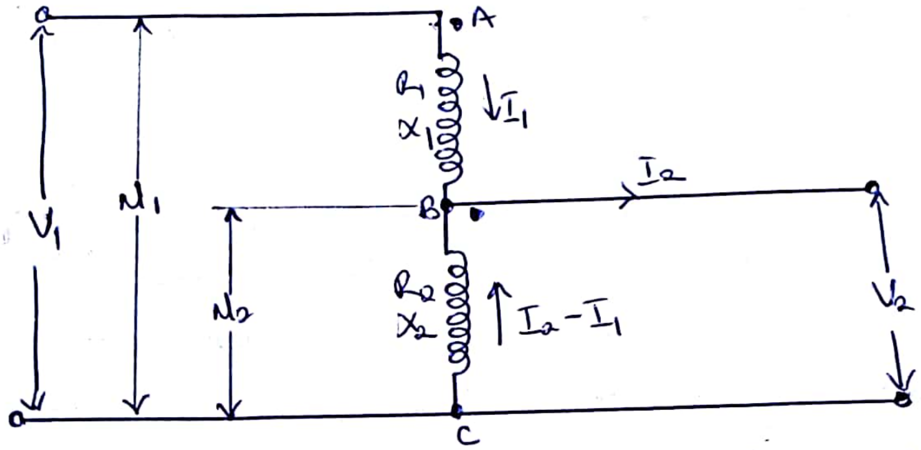


Fig 2.17: Autotransformer connection in step down mode

From the figure,

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1 - N_2}{N_2} \; ; N_1 > N_2 \quad \text{--- } 2.52$$

turn-ratio of auto transformer is given by

$$a' = \frac{V_1}{V_2} = \frac{N_1}{N_2} > 1 \quad \text{--- } 2.53$$

from eqn 2.52 & 2.53

$$a' = 1 + a$$

Comparing the VA rating of an autotransformer with a two winding transformer,

$$(VA)_{TW} = (V_1 - V_2) I_1 = (I_2 - I_1) V_2 \quad \text{--- } 2.54$$

and

$$(VA)_{auto} = V_1 I_1 = V_2 I_2 \quad \text{--- } 2.55$$

then

$$(VA)_{TW} = \left(1 - \frac{V_2}{V_1}\right) V_1 I_1 = \left(1 - \frac{N_2}{N_1}\right) (VA)_{auto} \quad \text{--- } 2.56$$

or $(VA)_{auto} = \left[\frac{1}{1-(1/a')}\right](VA)_{TW}$; $a' = \frac{N_1}{N_2} > 1$ --- 2.57

It follows that

$$(VA)_{auto} > (VA)_{TW}$$

It is therefore seen that a two-winding transformer of a given VA rating when connected as an autotransformer can handle higher VA.

2.8: INSTRUMENT TRANSFORMERS

In power systems, two special-purpose transformers are used for taking measurements. These are the potential transformer and the current transformer.

2.8.1: POTENTIAL TRANSFORMER: This is a specially wound transformer with a high-voltage primary and a low-voltage secondary. It has a very low power rating, and it's sole purpose is to provide a sample of the power system's voltage to the instruments monitoring it. Since the pricipal purpose of the transformer is voltage sampling, it must be very accurate so as not to distort the true voltage values too badly. ~~Potential transformers of several~~

2.8.2 CURRENT TRANSFORMER: They sample the current in a line and reduce it to a safe and measurable level. A diagram of a typical current transformer is shown in fig 2.18 The current transformer consist of a secondary winding wrapped round a ferromagnetic ring, with the single primary line running through the center of the ring. The ferromagnetic ring holds and concentrates a small sample of the flux from the primary line. That flux then induces a voltage and current in the secondary windings.

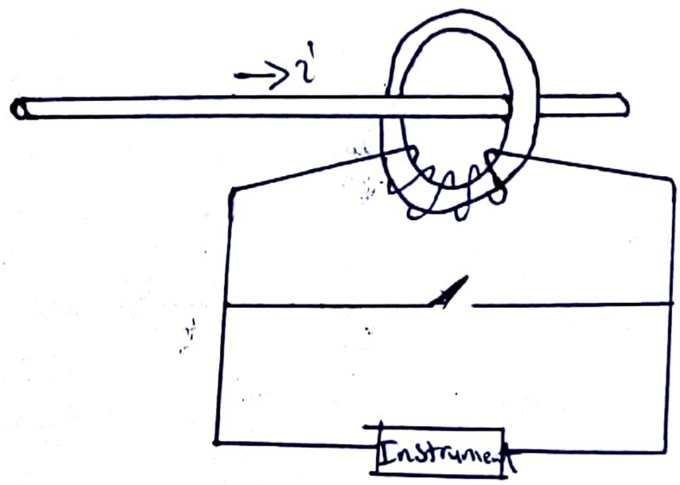


Fig 2.18 : Current trasformer

2.9 THREE-PHASE TRANSFORMERS

In generation, transformation, transmission and utilization of electric energy, it can be shown that it is economical to use the three-phase system rather than the single phase. It is on this premise that it become necessary to understand how transformers are used in them. In order to use a transformer in a three-phase circuit, ~~two~~ one of two configuration is possible: one is connecting a bank of three single-phase transformers and the other is a a single three-phase transformer with the primary and secondary of each phase wound on three legs of a common core. The construction of a single three-phase transformer is preferred in practice, since it is lighter, smaller, cheaper and slightly more efficient. The use of single-phase bank has the ~~the~~ advantage that each unit in the bank could be replaced individually when any becomes inoperative.

In a three-phase bank the phases are electrically connected but the three magnetic circuits are independent. ~~While~~ In the more common three-phase, 3-limb core-type transformer, the three magnetic circuits are also linked. Where delinking of the magnetic circuits is desired in a three-phase unit, a 5-limb shell type transformer could be used. Fig 2.19 illustrates the cores of the three-phase transformer.

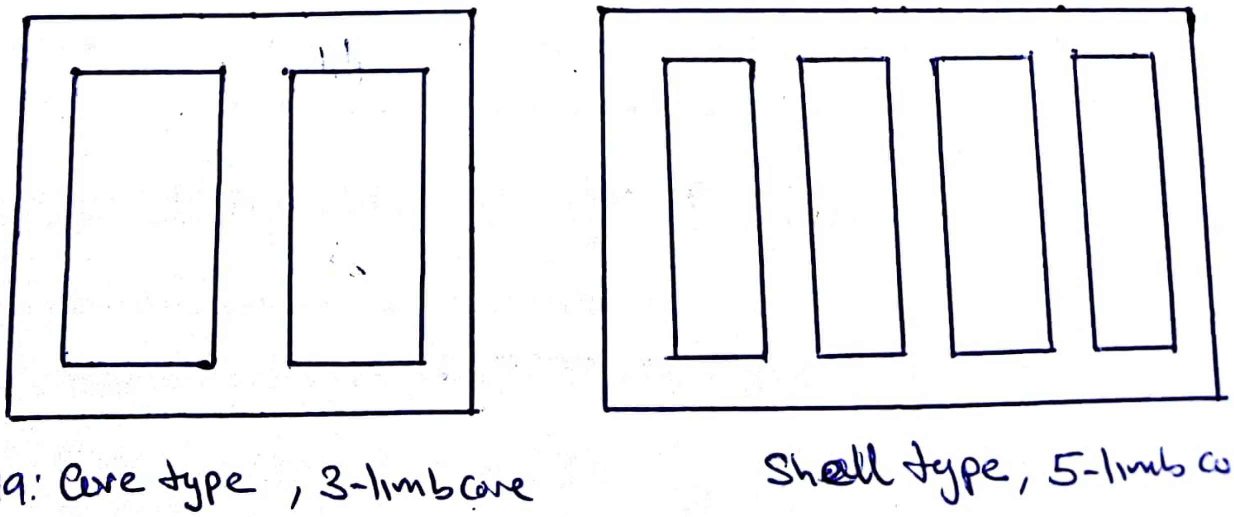
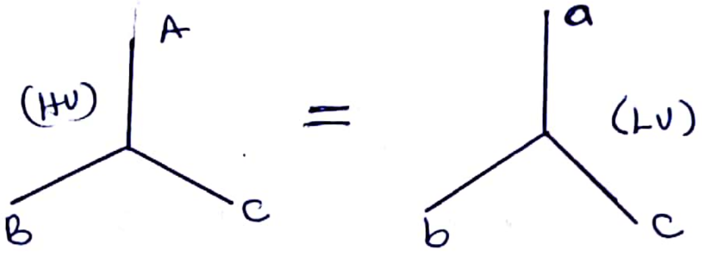


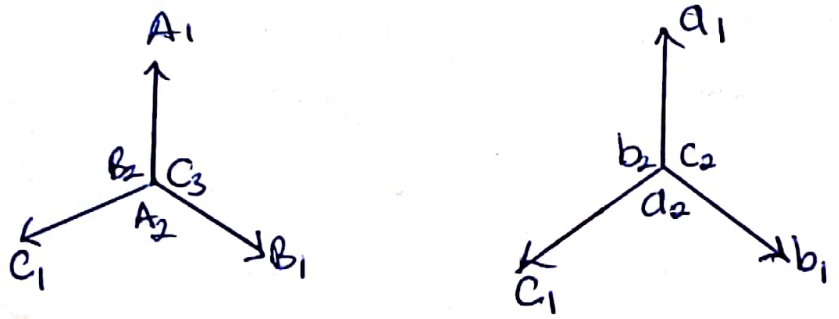
Fig 2.19: Core type, 3-limb core — Shell type, 5-limb core

2.9.1: STANDARD TERMINAL MARKINGS

Capital letters A, B, C are used for HV side and those on the LV side are labelled as small letters a, b, c.



Where a tertiary winding is available 3A, 3B, 3C are used. To mark the end of each winding, numbers suffixes 1 and 2 are used with 1's indicating similar polarity ends and so do 2's. For example



2.9.2: THREE-PHASE TRANSFORMER CONNECTIONS

A number of connections are possible on each side of a 3-phase transformer (single unit or bank). The primary and secondaries may be independently connected ie either a Star/wye (Y) or a delta (Δ). This leads to four possible connection of a three phase transformer:

1. Star – Star/wye-wye (Y–Y)
2. Star/wye – Delta (Y–Δ)
3. Delta – Star/wye (Δ–Y)
4. Delta – delta (Δ – Δ)

Star-Star | Wye-Wye Connection:

This is formed on each side of a 3-phase transformer by connecting together phase windings terminals suffixes marked 1 as in fig 2:20. In Y-Y connection, the primary voltage on each phase of the transformer is given by $V_{\phi P} = V_{LP}/\sqrt{3}$. The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer. The phase voltage on the secondary is then related to the line voltage on the secondary by $V_{LS} = \sqrt{3}V_{\phi S}$. Therefore, ~~overall~~ the voltage ratio on the transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\, V_{\phi P}}{\sqrt{3}\, V_{\phi S}} = a$$

This shows that if the phase transformation ratio is a:1, the line transformation (line-to-line voltages, line currents) ratio is also a:1.

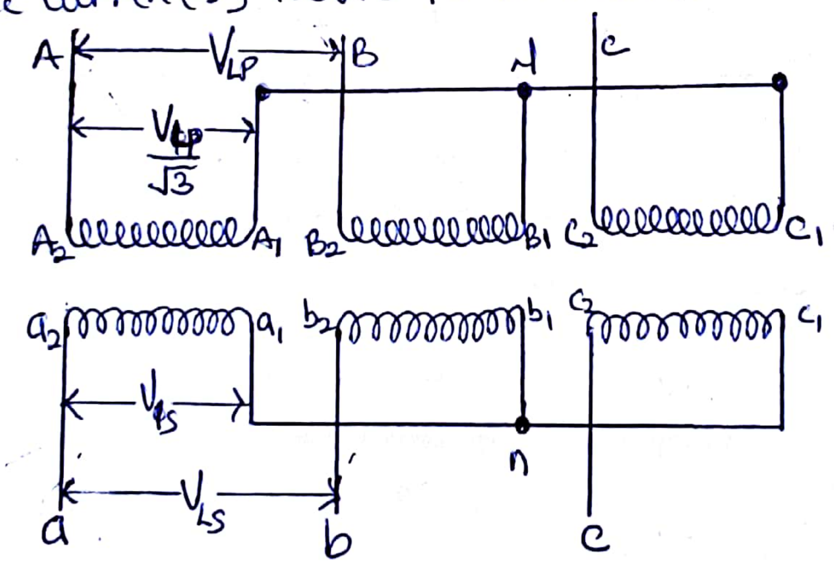


Fig 2:20 a

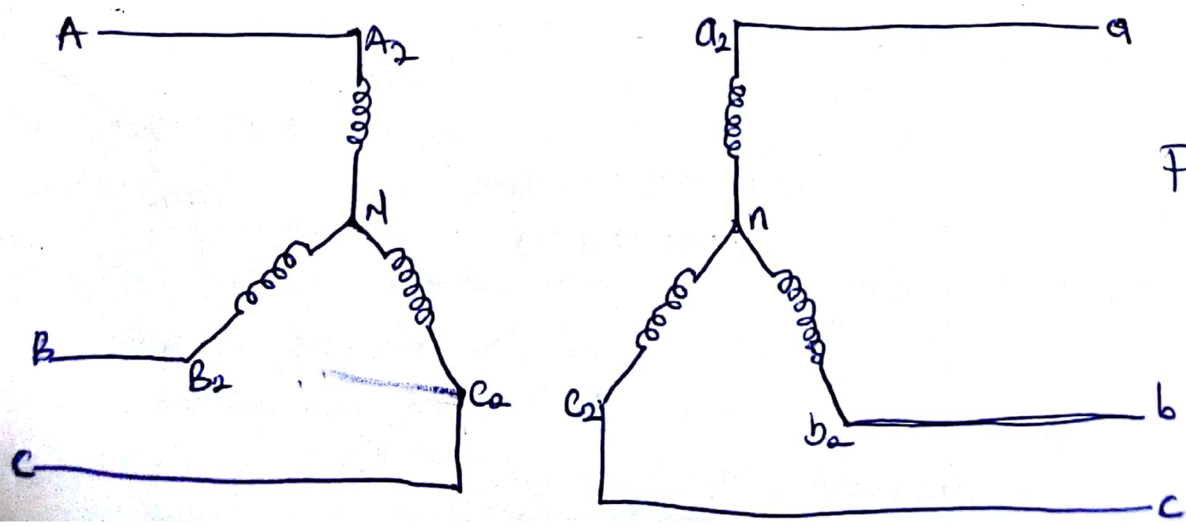


Fig 2:20 b

This connection is most economical for small, high voltage transformers because the number of turns/phase and the amount of insulation required is minimum.

Star/wye – Delta Connection.

This connection is formed on the primary side by connecting together 1 suffixed terminals; 2 suffixed terminals being connected to appropriate lines: the delta is formed by connecting c_1a_2, a_1b_2 and b_1c_2 with lines connected to these junctions being labelled a, b, and c respectively as shown in fig 2:21 The relationship between the primary line ~~phase~~ voltage and the primary phase voltage is given by $V_{LP} = \sqrt{3}\, V_{\phi P}$, while the secondary line voltage, is equal to the secondary phase voltage $V_{LS} = V_{\phi S}$.

Overall relationship between the line voltage on the primary side of the transformer and ~~to~~ the line voltage on the secondary side is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\, V_{\phi P}}{V_{\phi S}} = \sqrt{3}\, a$$

It therefore follows that if the phase transformation ratio of the star/delta connection is a:1, the line transformation ratio in magnitude is $\sqrt{3}a : 1$.

Because of this connection, the secondary voltage is shifted 30° relative to the primary voltage of the transformer. This means that a Y – Δ transformer bank cannot be paralleled with either a Y – Y or Δ – Δ bank. The connection shown in fig is known as the −30°-connection ~~and cause the seconde~~ ie the secondary voltage is lagging with a system phase sequence of abc. If the system phase sequence is acb, then the connection will cause the secondary voltage to be leading the primary by 30°. This connection finds application at the substation end of the transmission line where the voltage is to be stepped down.

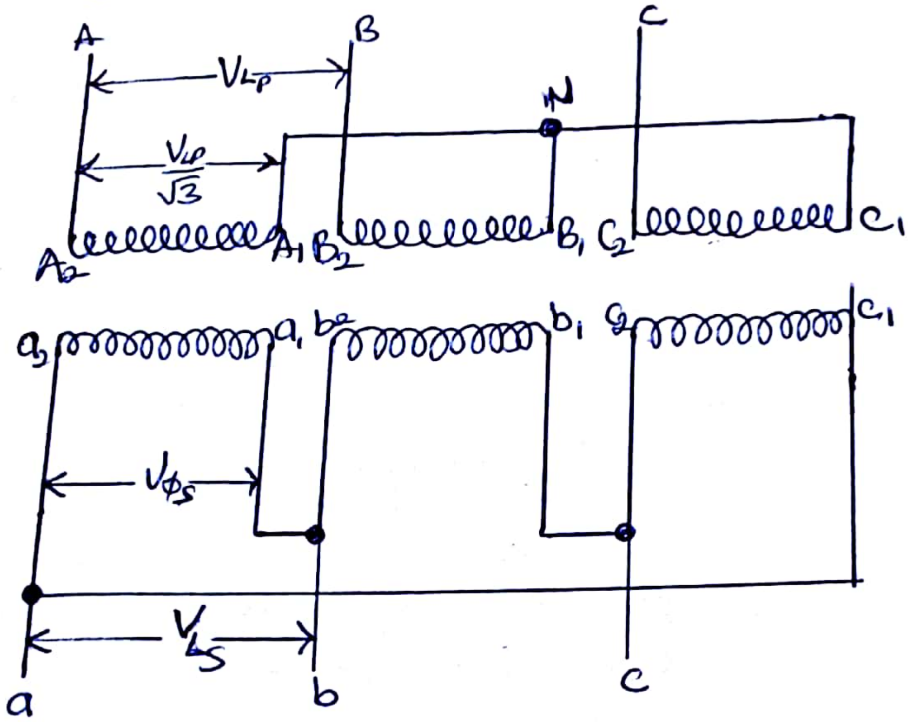


Fig: 2:21a

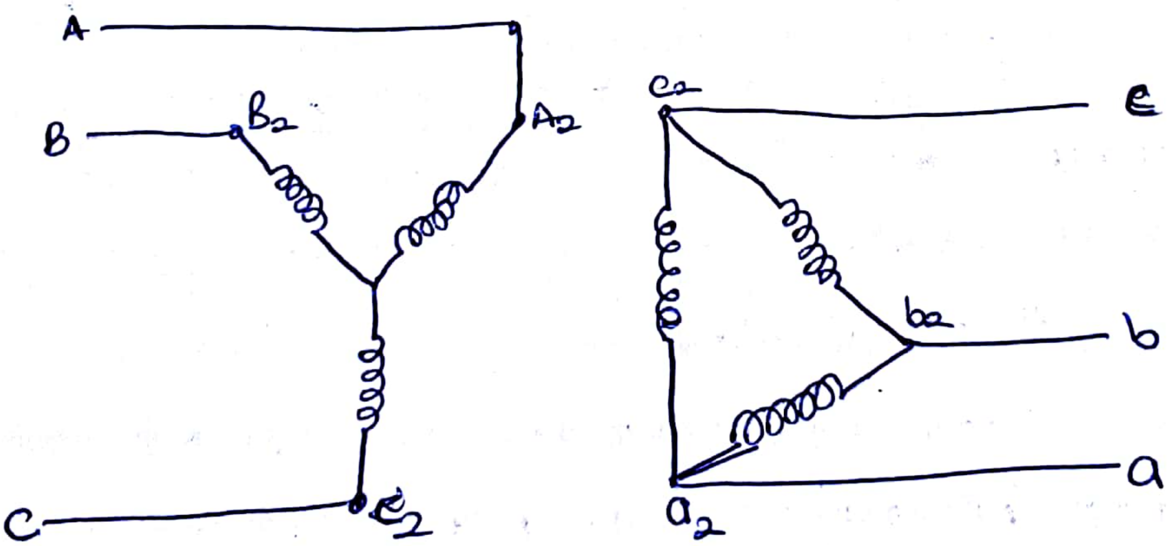


Fig 2:21b

Delta-Star/wye Connection $\Delta-Y$

This connection is simply the interchange of primary and secondary roles in the Star/wye-delta connection. In a $\Delta-Y$ connection, the primary line voltage is equal to the primary phase voltage $V_{LP} = V_{\phi P}$, while the secondary voltages are related by ~~the~~ $V_{LS} = \sqrt{3} V_{\phi S}$. Thus line-to-line voltage ratio of this connection is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3} V_{\phi S}} = \cancel{\frac{\sqrt{3}}{a}} \frac{a}{\sqrt{3}}$$

Thus, if the phase transformation ratio is $a:1$ (delta/star), the transformation ratio for line quantities will be $(a/\sqrt{3}):1$.

~~Contrary to~~ In contrast to the Star-delta connection -30° connection will now be the $+30^\circ$ - connection and vice versa if the capital and small letter suffixes of fig 2.21 are interchanged. ~~Thus~~ In addition, this connection is employed where it is necessary to step up the voltages.

Delta-Delta Connection $\Delta-\Delta$

Figure 2.22 Shows the delta-delta connection. The sum of voltages around the secondary delta must be zero; otherwise delta, being a closed circuit, means short circuit. In a $\Delta-\Delta$ connection, $V_{LP} = V_{\phi P}$ and $V_{LS} = V_{\phi S}$, so the relationship between primary and secondary line voltage is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a$$

This transformer has no phase-shift and if the phase transformation ratio is $a:1$, the transformation ratio for line quantities is also $a:1$. This connection is economical long, low-voltage transformers. In which insulation problem is not so urgent, because it increases the number of turns/phase.

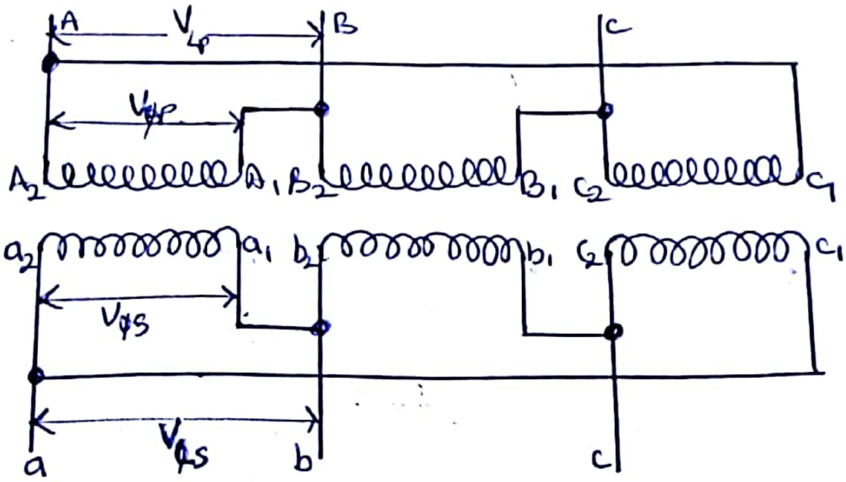


Fig 2.22 a

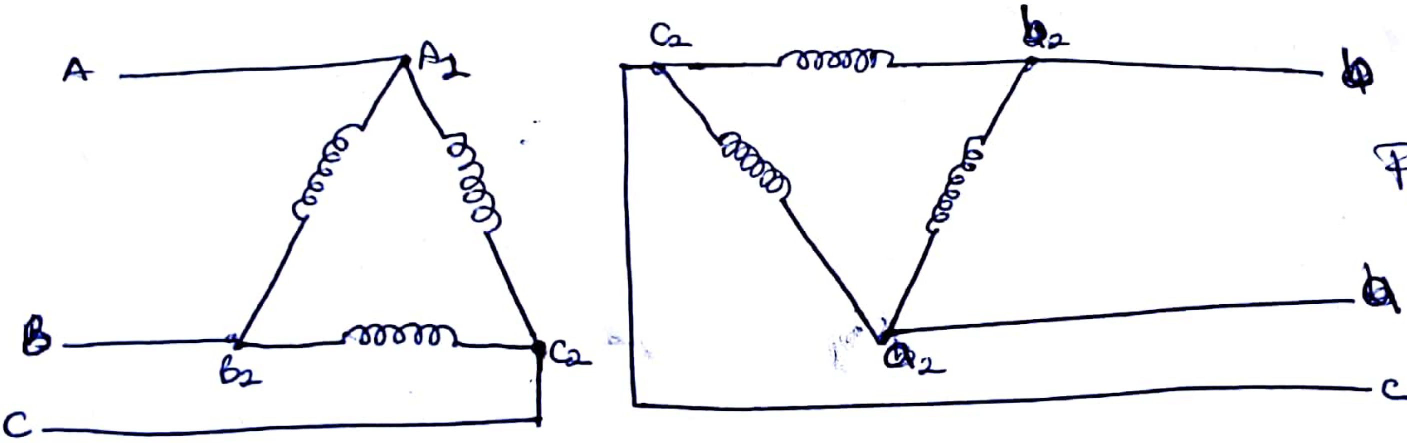


Fig 2.22b